MAC2312 Test 1 Name Key

Differentiate each of the following. **Do not simplify.**

1. $y = \sinh(3x)\ln(4x+1)$

$$\boxed{y' = \sinh(3x) \cdot \frac{1}{4x+1} \cdot 4 + \ln(4x+1) \cdot \cosh(3x) \cdot 3}$$

2. $y = tanh^{-1}(3x^2)$

$$\boxed{y' = \frac{1}{1-(3x^2)^2} \cdot 6x}$$

3. $y = \frac{cos^{-1}(5x)}{\sqrt{x+2}}$

$$\boxed{y' = \frac{\sqrt{x+2} \cdot \left(\frac{-1}{\sqrt{1-(5x)^2}} \cdot 5\right) - \frac{\cos^{-1}(5x)}{2\sqrt{x+2}}}{\left(\sqrt{x+2}\right)^2}}$$

4. $y = (e^{3x-4} - 3^{\pi})^4$

$$\boxed{y' = 4\left(e^{3x-4} - 3^{\pi}\right)^3 \left(e^{3x-4} \cdot 3 - 0\right)}$$

5. $y = \left(1 + \frac{4}{x}\right)^x$

$$\ln y = \ln\left(1 + \frac{4}{x}\right)^x$$

$$\ln y = x \cdot \ln\left(1 + \frac{4}{x}\right)$$

$$\frac{1}{y} \cdot y' = x \cdot \left(\frac{1}{1+\frac{4}{x}} \cdot \frac{-4}{x^2}\right) + \ln\left(1 + \frac{4}{x}\right) \cdot 1$$

$$\boxed{y' = \left(1 + \frac{4}{x}\right)^x \left[x \cdot \left(\frac{1}{1+\frac{4}{x}} \cdot \frac{-4}{x^2}\right) + \ln\left(1 + \frac{4}{x}\right)\right]}$$

Calculate the limits of each of the following.

6. $\lim_{x\to 3^-}\left(e^{\left(\frac{2}{x-3}\right)}\right) = e^{\frac{2}{3^- - 3}} = e^{\frac{2}{0^-}} = e^{-\infty} = \boxed{0}$

7. $\lim_{x\to 0}\left(\frac{1-\cos(x)}{x^2+x}\right) = \frac{1-\cos(0)}{0^2+0} = \frac{0}{0}$

L $\lim_{x\to 0} \frac{\sin(x)}{2x+1} = \frac{\sin(0)}{2(0)+1} = \frac{0}{1} = \boxed{0}$

8. $\lim_{x\to 0}\left(\frac{\tan(\pi x)}{\ln(1+x)}\right) = \frac{\tan(\pi\cdot 0)}{\ln(1+0)} = \frac{0}{0}$

L $\lim_{x\to 0} \frac{\pi \sec^2(\pi x)}{\frac{1}{1+x}} = \frac{\pi \sec^2(\pi\cdot 0)}{\frac{1}{1+0}} = \frac{\pi(1)}{1} = \boxed{\pi}$

9. $\lim_{x\to 0}\left(\frac{\sinh(x)}{x+1}\right) = \frac{\sinh(0)}{0+1} = \frac{0}{1} = \boxed{0}$

10. $\lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n = L$

$$\lim_{n\to\infty} \ln\left(1+\frac{1}{n}\right)^n = \ln L$$

$$\lim_{n\to\infty} n\cdot \ln\left(1+\frac{1}{n}\right) = \ln L$$

2

$$\lim_{n\to\infty} \frac{\ln\left(1+\frac{1}{n}\right)}{\frac{1}{n}} = \ln L$$

2 $$\lim_{n\to\infty} \frac{\frac{1}{1+\frac{1}{n}}\cdot\frac{-1}{\cancel{n^2}}}{\frac{-1}{\cancel{n^2}}} = \ln L$$ 2

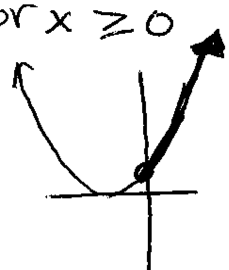
$$\lim_{n\to\infty} \frac{1}{1+\frac{1}{n}} = \ln L$$

2

$$\frac{1}{1+\frac{1}{\infty}} = 1 = \ln L \qquad \boxed{L = e}$$ 2

$$e^1 = L$$

11. Find $(f^{-1})'(1)$ if $f(x) = x + x^2 + e^x$. for $x \geq 0$



$f'(x) = 1 + 2x + e^x$

* $f(x)$ is differentiable

2

Use Theorem 7 :

$$(f^{-1})'(1) = \left.\frac{1}{f'(x)}\right|_{x=f^{-1}(1)}$$

$$= \left.\frac{1}{1+2x+e^x}\right|_{x=0}$$ 2

$$= \frac{1}{1+2\cdot 0+e^0} = \boxed{\frac{1}{2}}$$ 2

$f(x)$ is one-to-one (passes HLT) for $x \geq 0$ 2 *

$1 = x + x^2 + e^x$

$x = 0$

$f^{-1}(1) = 0$